LEVEL SET SEGMENTATION WITH ROBUST IMAGE GRADIENT ENERGY AND STATISTICAL SHAPE PRIOR

S. Y. Yeo, X. Xie, I. Sazonov, P. Nithiarasu

Swansea University, Singleton Park, Swansea, SA2 8PP, UK
{465186, x.xie, i.sazonov, p.nithiarasu}@swansea.ac.uk

ABSTRACT

We propose a new level set segmentation method with statistical shape prior using a variational approach. The image energy is derived from a robust image gradient feature. This gives the active contour a global representation of the geometric configuration, making it more robust to image noise, weak edges and initial configurations. Statistical shape information is incorporated using nonparametric shape density distribution, which allows the model to handle relatively large shape variations. Comparative examples using both synthetic and real images show the robustness and efficiency of the proposed method.

***Index Terms*—** Level set, shape prior, energy minimization, variational method

1. INTRODUCTION

Active contours provide an effective framework for object segmentation as they can easily adapt to shape variations. Various types of information can also be incorporated to regularize the smoothness and shape of the contour. However, it is still a great challenge for active contour models to achieve strong initialization invariancy and robust convergence. This is particularly true when the active contour is applied on real image dataset consisting of varying intensities and complex geometries. In the presence of artifacts, occlusions or large amount of noise, it is difficult for purely image-based models to extract objects accurately. In such cases, prior knowledge of shape information can be very useful. Many shape prior techniques are based on simplistic statistical assumptions, such as the training shapes are constrained to a Gaussian distribution. However, real world objects often exhibit complex shape variations and the projection from 3D object to 2D image can be nonlinear. Nonparametric technique, such as [1, 2, 3], models the shape distribution by applying the kernel density estimation (KDE) in a shape space. This allows the model to handle relatively large shape variations.

In this paper, we present a variational level set model using Bayesian inference, which decomposes the associated cost function into two parts: image energy part and shape energy part. Image intensity/color or its local distribution has been commonly used to derive the image energy, such as [1, 3]. Texture information, e.g. derived from filter banks, may also be used. However, they may result in a large dimensional feature space, which can be difficult to formulate in level set framework without cascading the feature vectors that may reduce its discriminability. Image gradient is susceptible to image noise and weak edges as it is a local measure, and is generally abandoned in region based models. In this work, we use an image gradient feature which is computed based on global interaction among gradient vectors. It has been shown in [4, 5] that its vector form can directly drive active contours much more efficiently and effectively in terms of handling image noise, challenging initialization, weak edge and even broken object boundaries. Here, we use its scalar form as an image feature to indicate the presence of object boundaries. Its characteristics are fundamentally different from image intensity or image gradient iteself. The shape energy is based on nonparametric shape distribution, which is introduced in [1]. The use of nonparametric technique of kernel density estimation (KDE) allows the shape prior to approximate arbitrary distributions, and hence may handle large shape variations in the training set. This combination of image energy and shape energy allows the proposed model conveniently handle feature inhomogeneity, noise, and occlusion.

2. PROPOSED METHOD

Briefly, the proposed model consists of an image attraction force which propagates contours towards object boundaries, and a global shape force which deforms the model according to the shape distribution learned from a training set. The image attraction force is derived from the interaction of gradient vectors. It differs from conventional image gradient based methods as it utilizes pixels interactions across the whole image domain. A shape distance is defined to measure the dissimilarity between shapes. The statistical shape information is incorporated into the model using nonparametric shape density distribution of the training shapes.

2.1. Bayesian formulation of segmentation model

In this section, we formulate the segmentation model using Bayesian inference, where the segmentation of an image I

can be considered as maximizing the conditional probability given as

$$p(\phi|I) = \frac{p(I|\phi)p(\phi)}{p(I)} \tag{1}$$

The shape that maximizes the posterior probability distribution can be estimated using a maximum a posteriori (MAP) approach:

$$\begin{aligned}\hat{\phi} &= \underset{\phi}{\operatorname{argmax}}\, p(\phi|I) \propto \underset{\phi}{\operatorname{argmax}}\, p(I|\phi)\cdot p(\phi)\\ &= \underset{\phi}{\operatorname{argmin}} \Big(-log(p(I|\phi)) - log(p(\phi))\Big)\end{aligned} \tag{2}$$

since $p(I)$ is independent of the shape ϕ and is constant for a given image. The MAP estimation of the shape in (2) that maximizes the posterior probability can also be achieved to minimize the following energy functional:

$$\begin{aligned}E(\phi) &= -log(p(I|\phi)) - log(p(\phi))\\ &= E_{image}(\phi) + \alpha E_{shape}(\phi)\end{aligned} \tag{3}$$

where $E_{image}(\phi)$ represents the image based term, $E_{shape}(\phi)$ represents the shape prior and α is a constant.

2.2. Image based energy

The image based term is used to propagate the model towards the feature of interest in the image and can be image gradient based or region based. Conventional image gradient based methods [6, 7] are often sensitive to image noise as they make use of local image information. The gradient vector flow (GVF) model in [8] uses vector diffusion which increases the attraction range and allows the model to handle boundary concavities. It however has convergence issues caused by saddle or stationary points in its force field. Although region based techniques [9, 10, 11] exhibit more robustness against noise, they often cannot handle feature inhomogeneity. In [5], we derived a new image attraction force based on hypothesized gradient vector interactions [5] for contour evolution, which can be considered as a generalization of the MAC model [4] in 2D. Here, we formulate it in a variational framework so that statistical prior information can be conveniently incorporated into the model.

We propose the following image based energy functional:

$$E_{image}(\phi) = \nu \int_{\Omega} g(\mathbf{x})|\nabla H(\phi)|d\mathbf{x} + \int_{\Omega} G(\mathbf{x})H(\phi)d\mathbf{x} \tag{4}$$

where ν is a constant parameter, $g(\mathbf{x}) = 1/|1 + \nabla I|$, and H is the Heaviside function. $G(\mathbf{x})$ represents the gradient vector interaction field given as:

$$G(\mathbf{x}) = \int_{\Omega} \frac{\hat{\mathbf{r}}_{\mathbf{x}\mathbf{x}'}}{r^k_{\mathbf{x}\mathbf{x}'}} \cdot \nabla I(\mathbf{x})d\mathbf{x} \tag{5}$$

where $\hat{\mathbf{r}}_{\mathbf{x}\mathbf{x}'}$ is the unit vector from pixel location $\mathbf{x}$ to $\mathbf{x}'$ and $r_{\mathbf{x}\mathbf{x}'}$ is the distance between the pixels. k is a constant which coincides with the dimension of the image data (i.e. $k = 2$ for 2D image). The first term in (4) induces the segmentation model to favor minimal length, while the second term attracts the active contour towards object boundaries

Although the gradient vector interaction field $G(\mathbf{x})$ is derived from image gradients, it utilizes image pixels or voxels across the whole image domain, and thus gives a global representation of the geometric configuration. This provides the active contour a high invariancy to initializations and a large attraction range. It also increases the robustness of the active contour against image noise.

2.3. Shape based energy

We define the shape energy functional using a shape distance measure proposed in [1] as:

$$\begin{aligned}E_{shape}(\phi) &= D^2(\phi, \phi_i)\\ &= \int_{\Omega} (H(\phi(\mathbf{x} + \mu_\phi) - H(\phi_i))^2 d\mathbf{x}\end{aligned} \tag{6}$$

where $\{\phi_i\}_{i=1...N}$ is a set of training shapes, and μ_ϕ is the center of gravity of the shape ϕ. Note that the training shapes ϕ_i are assumed to be aligned with respect to their center of gravity. The intrinsic alignment in the shape distance provides a dissimilarity measure which is invariant to the location of the shape ϕ.

Here, we use the nonparametric technique of KDE to model the statistical shape distribution:

$$p(\phi) \propto \frac{1}{N}\sum_{i=1}^{N} exp\left(-\frac{1}{2\sigma^2}D^2(\phi, \phi_i)\right) \tag{7}$$

where σ is the kernel width, and is set to the mean nearest-neighbor distance. The shape prior is invariant to the translation of the shape ϕ. Intrinsic alignments with respect to scale and rotation can also be incorporated in the model [1].

2.4. Level set based energy minimization

The minimization of the energy functional in (3) generates a segmentation model which attracts the active contour towards image object boundaries under combined influence from image feature and shape prior. It can be realized using variational gradient descent:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E_{image}(\phi)}{\partial \phi} - \alpha\frac{\partial E_{shape}(\phi)}{\partial \phi} \tag{8}$$

The image based gradient flow is given as:

$$\frac{\partial E_{image}(\phi)}{\partial \phi} = \nu g(\mathbf{x})\nabla\cdot\Big(\frac{\nabla\phi(\mathbf{x})}{|\phi(\mathbf{x})|}\Big)\delta(\phi(\mathbf{x})) - G(\mathbf{x})\delta(\phi(\mathbf{x})) \tag{9}$$

and the shape gradient flow is defined as:

$$\frac{\partial E_{shape}(\phi)}{\partial \phi} = \frac{\sum_i w_i \frac{\partial D^2(\phi,\phi_i)}{\partial \phi}}{2\sigma^2 \sum_i w_i} \tag{10}$$

with shape force weighted by the factor:

$$w_i = exp\Big(-\frac{1}{2\sigma^2} D^2(\phi, \phi_i)\Big) \tag{11}$$

and the shape derivative with respect to ϕ is given as:

$$\begin{aligned} &\frac{\partial D^2(\phi, \phi_i)}{\partial \phi} \\ &= 2\delta(\phi(\mathbf{x}))\Big((H(\phi(\mathbf{x})) - H(\phi_i(\mathbf{x} - \mu_\phi)) + \frac{(\mathbf{x} - \mu_\phi)^T}{\int_\Omega H(\phi) d\mathbf{x}} \\ &\times \int (H(\phi(\mathbf{x}') - H(\phi_i(\mathbf{x}' - \mu_\phi)\delta(\phi(\mathbf{x}'))\nabla\phi(\mathbf{x}') d\mathbf{x}'\Big) \end{aligned} \tag{12}$$

3. IMPLEMENTATION

The image object boundary representation used in the derivation of $G(\mathbf{x})$ can be computed using central differences, or standard edge detection methods such as the Sobel filter. Some effects caused by spurious edges can be removed by not considering pixels with very small edge magnitude, i.e. 5% - 10% of the maximum magnitude. $G(\mathbf{x})$ can be computed efficiently as a vector convolution using fast Fourier transform (FFT).

The Heaviside function H in (4) is approximated by the regularized function H_ϵ defined as:

$$H_\epsilon(x) = \frac{1}{2}\Big(1 + \frac{2}{\pi} arctan(\frac{x}{\epsilon})\Big) \tag{13}$$

and the Dirac delta function δ in (6) is approximated by the derivative of H_ϵ as:

$$\delta_\epsilon(x) = \frac{\epsilon}{\pi(\epsilon^2 + x^2)} \tag{14}$$

where ϵ is an arbitrary small constant. The finite difference method is used to approximate the derivatives, and the narrow band approach is used to reduce the computational cost in updating the level set function.

4. RESULTS

In this section, we show that the proposed method can be applied to efficiently segment image objects. The proposed method was compared against edge based method without shape prior [5, 4], Chan-Vese (CV) region based model [9], and CV with shape prior on both synthetic and real images.

Figure 1 depicts the segmentation of multiple annular-like objects from an image with 40% noise and intensity variation. It is shown that the image based energy derived from the global interactions of gradient vectors is robust to image noise and allows the active contour to extract the shapes accurately with an arbitrary cross-boundary initialization. Although the Chan and Vese model can handle the image noise, it cannot deal with the inhomogeneous intensity as shown in the figure.

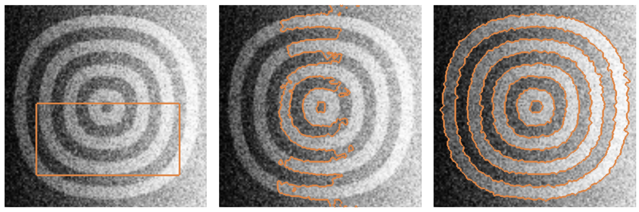

Fig. 1. Segmentation of annular-like shapes from noisy image: (from left to right) initial contour, region based energy [9], gradient vector interaction based energy.

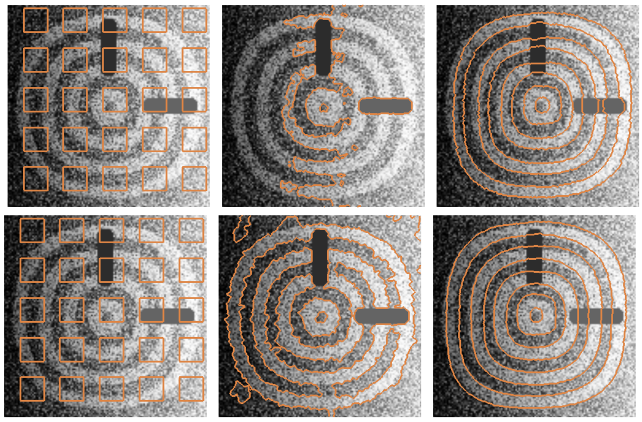

Fig. 2. Segmentation of annular-like shapes from occluded and noisy image: (top row, from left to right) initial contour, CV, CV with shape prior, (bottom row, from left to right) initial contour, gradient vector interaction based energy, gradient vector interaction and shape based energy.

Next, we consider a training set of 20 images with annular-like objects of considerable shape variations. The shape prior is incorporated to the active contour models to extract the shapes from an image with 70% noise, occlusions and intensity variation. As shown in the top row of Figure 2, the shape force overwhelmed the region based force in the Chan and Vese model [9], and did not localize on the boundaries of the objects accurately. In contrast, the proposed active contour with shape prior extracted the shapes efficiently.

We also apply the proposed level set segmentation with shape prior on real images. In each of the examples, 15 training shapes are used to model the shape distribution. The statistical shape prior is then used to segment images with shapes not included in the training set. Figure 3 depicts the segmentation of the knee from MR image. It is shown that the gradient vector interaction based energy provides a more accurate segmentation of the knee as compared to the region based energy. The incorporated shape prior information also performs

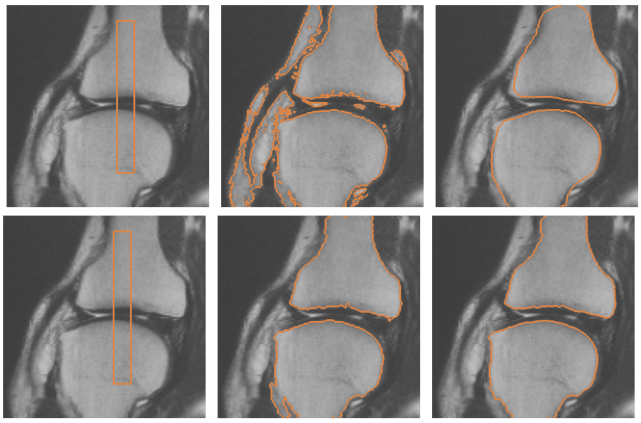

Fig. 3. Segmentation of the knee from MR image: (top row, from left to right) initial contour, CV, CV with shape prior, (bottom row, from left to right) initial contour, gradient vector interaction based energy, gradient vector interaction and shape based energy.

better with the proposed method.

The segmentation of corpus callosum from MR image are depicted in Figure 4. It is shown that the gradient vector interaction model is more efficient than the region based model in segmenting the brain image. The proposed model with shape prior also provides a more accurate segmentation than the region based model with shape prior. Figure 5 shows another example in which the proposed method efficiently segment the shape from the image.

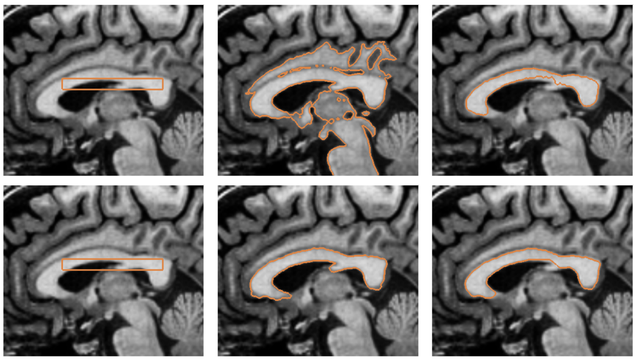

Fig. 4. Segmentation of corpus callosum from MR image: (top row, from left to right) initial contour, CV, CV with shape prior, (bottom row, from left to right) initial contour, gradient vector interaction based energy, gradient vector interaction and shape based energy.

5. CONCLUSION

We have presented a new variational model for level set segmentation with statistical shape prior. The image based energy derived from the global interaction of gradient vectors provides a more coherent and global representation of the geometric configuration. The active contour model is thus more robust to image noise and weak edges, and has a strong invariancy to initializations. By using kernel density estimation, the incorporated shape prior can model arbitrary shape distributions. The proposed model can thus segment complex shapes from occluded and noisy images efficiently.

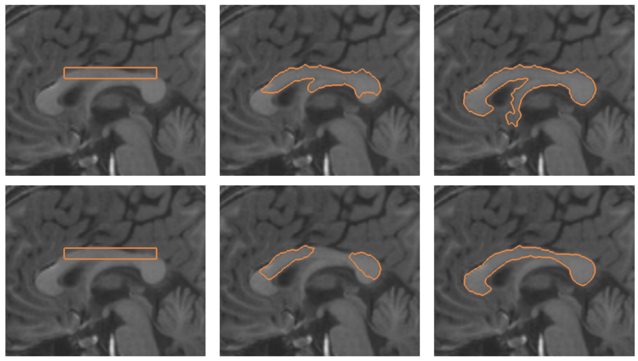

Fig. 5. Segmentation of corpus callosum from MR image using the proposed active contour: (top row) gradient vector interaction based energy, (bottom row) gradient vector interaction and shape based energy.

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